

INTEGRAL CHARACTERISTICS OF BREMSSTRAHLUNG AND PAIR PHOTOPRODUCTION IN A MEDIUM

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The bremsstrahlung of an electron and e^-e^+ -pair creation by a photon in a medium is considered in high-energy region, where influence of the multiple scattering on the processes (the Landau-Pomeranchuk-Migdal (LPM) effect) becomes essential. The integral characteristics: the radiation length and the total probability of radiation and pair photoproduction are analyzed under influence of the LPM effect.

1 Introduction

When a charged particle is moving in a medium it scatters on atoms. With probability $\sim \alpha$ this scattering is accompanied by a radiation. At high energy the radiation process occurs over a rather long distance, known as the *formation length* l_c :

$$l_c = \frac{l_0}{1 + \gamma^2 \vartheta_c^2}, \quad l_0 = \frac{2\varepsilon\varepsilon'}{m^2\omega}, \quad (1)$$

where ω is the energy of emitted photon, $\varepsilon(m)$ is the energy (the mass) of a particle, $\gamma = \varepsilon/m$ is the Lorenz factor, $\varepsilon' = \varepsilon - \omega$, ϑ_c is the characteristic angle of photon emission, the system $\hbar = c = 1$ is used.

Landau and Pomeranchuk were the first who showed that if the formation length of bremsstrahlung becomes comparable to the distance over which the multiple scattering becomes important (when the mean angle of multiple scattering is of the order of the characteristic angle of photon emission $\sim 1/\gamma$), the bremsstrahlung will be suppressed¹. Migdal² developed the quantitative theory of this phenomenon.

New activity with the theory of the LPM effect (see^{3, 4, 5}) is connected with a very successful series of experiments performed at SLAC recently (see^{6, 7}). In these experiments the cross section of the bremsstrahlung of soft photons with energy from 200 keV to 500 MeV from electrons with energy 8 GeV and 25 GeV is measured with an accuracy of the order of a few percent. Both LPM and dielectric suppression are observed and investigated. These experiments were the challenge for the theory since in all the mentioned papers calculations are performed to logarithmic accuracy which is not enough for description of the new experiment. The contribution of the Coulomb cor-

rections (at least for heavy elements) is larger than experimental errors and these corrections should be taken into account.

We developed the new approach to the theory of the Landau-Pomeranchuk-Migdal (LPM) effect⁸ basing on the quasiclassical operator approach⁹. In this paper the cross section of the bremsstrahlung process in the photon energies region where the influence of the LPM is very strong was calculated with a term $\propto 1/L$, where L is characteristic logarithm of the problem, and with the Coulomb corrections taken into account. In the photon energy region, where the LPM effect is "turned off", the obtained cross section gives the exact Bethe-Maximon cross section (within power accuracy) with the Coulomb corrections. This important feature was absent in the previous calculations. Some important features of the LPM effect were considered also in^{10, 11, 12, 13}.

The crossing process for the bremsstrahlung is the pair creation by a photon. The created particles undergo here the multiple scattering. It should be emphasized that for the bremsstrahlung the formation length (1) increases strongly if $\omega \ll \varepsilon$. Just because of this the LPM effect was investigated at SLAC at a relatively low energy. For the pair creation by a photon with energy ω the formation length $l_p = \frac{2\varepsilon(\omega - \varepsilon)}{m^2\omega}$ attains maximum at $\varepsilon = \omega/2$ and this maximum is $l_{p,max} = (\omega/2m)\lambda_c$. Because of this even for heavy elements the effect of multiple scattering becomes noticeable at photon energies $\omega \geq 10$ TeV. Starting from these energies one has to take into account the influence of a medium on the pair creation and on the bremsstrahlung hard part of the spectrum in electromagnetic showers being created by the cosmic ray particles of the ultrahigh energies. These effects can be quite significant in the electromagnetic calorimeters operating in the detectors on the colliders in TeV range.

In the present paper the radiation length is calculated under influence of the LPM effect. The total probability of photon radiation and the integral probability of the pair creation are considered also.

2 Influence of the multiple scattering on the bremsstrahlung

2.1 Bremsstrahlung spectrum at high energy

The spectral radiation intensity obtained in⁸ (see Eq.(2.39)) has the form

$$dI = \omega dW = \frac{\alpha m^2 x dx}{2\pi(1-x)} \text{Im} \left[\Phi(\nu) - \frac{1}{2L_c} F(\nu) \right], \quad x = \frac{\omega}{\varepsilon}, \quad (2)$$

where

$$\begin{aligned}
\Phi(\nu) &= \int_0^\infty dz e^{-it} \left[r_1 \left(\frac{1}{\sinh z} - \frac{1}{z} \right) - i\nu r_2 \left(\frac{1}{\sinh^2 z} - \frac{1}{z^2} \right) \right] \\
&= r_1 \left(\ln p - \psi \left(p + \frac{1}{2} \right) \right) + r_2 \left(\psi(p) - \ln p + \frac{1}{2p} \right), \\
F(\nu) &= \int_0^\infty \frac{dz e^{-it}}{\sinh^2 z} [r_1 f_1(z) - 2ir_2 f_2(z)], \\
f_1(z) &= \left(\ln \varrho_c^2 + \ln \frac{\nu}{i} - \ln \sinh z - C \right) g(z) - 2 \cosh z G(z), \\
f_2(z) &= \frac{\nu}{\sinh z} \left(f_1(z) - \frac{g(z)}{2} \right), \quad g(z) = z \cosh z - \sinh z, \\
G(z) &= \int_0^z (1 - y \coth y) dy \\
&= z - \frac{z^2}{2} - \frac{\pi^2}{12} - z \ln(1 - e^{-2z}) + \frac{1}{2} \text{Li}_2(e^{-2z}), \\
t &= \frac{z}{\nu}, \quad r_1 = x^2, \quad r_2 = 1 + (1 - x)^2, \quad t = t_1 + t_2, \quad z = \nu t. \tag{3}
\end{aligned}$$

here $\alpha = 1/137$, $z = \nu t$, $p = i/(2\nu)$, $\psi(x)$ is the logarithmic derivative of the gamma function, $\text{Li}_2(x)$ is the Euler dilogarithm. Use of found form of Φ and the last representation of function $G(z)$ simplifies the numerical calculation. The term with $\Phi(\nu)$ in (2) describes the intensity in logarithmic approximation, the term with $F(\nu)$ is the first correction. The parameters in these formulas are

$$\begin{aligned}
\nu^2 &= i\nu_0^2, \quad \nu_0^2 = |\nu|^2 \simeq \nu_1^2 \left(1 + \frac{\ln \nu_1}{L_1} \vartheta(\nu_1 - 1) \right), \quad \nu_1^2 = \frac{\varepsilon}{\varepsilon_e} \frac{1-x}{x}, \\
\varepsilon_e &= m (8\pi Z^2 \alpha^2 n_a \lambda_c^3 L_1)^{-1}, \quad L_c \simeq L_1 \left(1 + \frac{\ln \nu_1}{L_1} \vartheta(\nu_1 - 1) \right), \quad L_1 = \ln \frac{a_{s2}^2}{\lambda_c^2}, \\
\frac{a_{s2}}{\lambda_c} &= 183 Z^{-1/3} e^{-f}, \quad f = f(Z\alpha) = (Z\alpha)^2 \sum_{k=1}^{\infty} \frac{1}{k(k^2 + (Z\alpha)^2)}, \tag{4}
\end{aligned}$$

here Z is the charge of the nucleus, n_a is the number density of atoms in the medium, $\lambda_c = 1/m$ is the electron Compton wavelength. The LPM effect manifests itself when

$$\nu_1(x_c) = 1, \quad x_c = \frac{\varepsilon}{\varepsilon_e + \varepsilon}. \tag{5}$$

In the case $\varepsilon \ll \varepsilon_e$ in the hard part of spectrum ($1 \geq x \gg x_c$) the parameter $\nu_1^2 \simeq x_c/x \ll 1$ and the contribution into the integral (3) give the

region $z \ll 1$.

$$\text{Im } \Phi(\nu) \simeq r_1 \frac{\nu_1^2}{6} + r_2 \frac{\nu_1^2}{3}, \quad -\text{Im } F(\nu) = -\frac{1}{9}(r_2 - r_1)\nu_1^2(1 + O(\nu_1^4)). \quad (6)$$

Substituting into (2) we have

$$\frac{dI}{dx} = \frac{2Z^2\alpha^3 n_a \varepsilon}{3m^2} \left[r_1 \left(L_1 - \frac{1}{3} \right) + 2r_2 \left(L_1 + \frac{1}{6} \right) \right] \quad (7)$$

This is the Bethe-Maximon intensity spectrum (with the Coulomb corrections) in case of complete screening (if one neglects the contribution of atomic electrons) written down within power accuracy (omitted terms are of the order of powers of $1/\gamma$), see e.g. Eq.(18.30) in ¹⁴. So, to obtain it in the limit considered one has to take into account the both terms in brackets in (2).

At very strong multiple scattering $\nu_0 \gg 1$ or $\varepsilon \gg \varepsilon_e$ one can omit e^{-it} in the integrand of function $F(\nu)$ (3). Integrating over z we obtain

$$-\text{Im } F(\nu) = \frac{\pi}{4}(r_1 - r_2) + \frac{\nu_0}{\sqrt{2}} \left(\ln 2 - C + \frac{\pi}{4} \right) r_2, \quad (8)$$

where we take into account the next terms of the decomposition in the term $\propto r_2$. Under the same conditions ($\nu_0 \gg 1$) the function $\text{Im } \Phi(\nu)$ is

$$\text{Im } \Phi(\nu) = \frac{\pi}{4}(r_1 - r_2) + \frac{\nu_0}{\sqrt{2}} r_2. \quad (9)$$

Thus, at $\nu_0 \gg 1$ the relative contribution of the first correction $\frac{dW^1}{d\omega}$ is defined by

$$r = \frac{dW^1}{dW^c} = \frac{1}{2L_c} \left(\ln 2 - C + \frac{\pi}{4} \right) \simeq \frac{0.451}{L_c}. \quad (10)$$

In the case $\varepsilon \geq \varepsilon_e$ the intensity spectrum differs from the Bethe-Maximon one at $x \sim 1$ also. When $\varepsilon \gg \varepsilon_e$ we find in the interval not very close to the end of the spectrum ($x = 1$):

$$\begin{aligned} \frac{dI}{dx} \simeq & \frac{2\sqrt{2}Z^2\alpha^3 n_a \varepsilon}{m^2} \sqrt{\frac{\varepsilon_e x}{\varepsilon(1-x)}} \left(1 + \frac{1}{4L_1} \ln \frac{\varepsilon(1-x)}{\varepsilon_e x} \right) \left[x^2 \right. \\ & \left. + 2(1-x) \left(1 - \frac{\pi}{2\sqrt{2}} \sqrt{\frac{\varepsilon_e x}{\varepsilon(1-x)}} \right) \right], \quad \varepsilon(1-x) \gg \varepsilon_e x. \end{aligned} \quad (11)$$

2.2 Integral characteristics of bremsstrahlung

Now we turn to the integral characteristics of radiation. The total intensity of radiation in the logarithmic approximation can be presented as (see (2))

$$\frac{I}{\varepsilon} L_{rad}^0 = 2 \frac{\varepsilon_e}{\varepsilon} \text{Im} \left[\int_0^1 \frac{dx}{g} \sqrt{\frac{x}{1-x}} (2(1-x) + x^2) \right. \\ \left. + \int_0^1 \frac{x^3 dx}{1-x} \left(\psi(p+1) - \psi\left(p+\frac{1}{2}\right) \right) + 2 \int_0^1 x dx (\psi(p+1) - \ln p) \right] \quad (12)$$

where

$$p = \frac{g\eta}{2}, \quad \eta = \sqrt{\frac{x}{1-x}}, \quad g = \exp\left(i\frac{\pi}{4}\right) \sqrt{\frac{L_1 \varepsilon_e}{L_c \epsilon}},$$

L_{rad}^0 is the radiation length in the logarithmic approximation. The relative energy losses of electron per unit time in terms of the Bethe-Maximon radiation length $\frac{I}{\varepsilon} L_{rad}^0$ in gold is given in Fig.1 (curve 1), it reduces by 10% (15% and 25%) at $\varepsilon \simeq 700$ GeV ($\varepsilon \simeq 1.4$ TeV and $\varepsilon \simeq 3.8$ TeV) respectively, and it cuts in half at $\omega \simeq 26$ TeV. This increase of effective radiation length can be important in electromagnetic calorimeters operating in detectors on colliders in TeV range. The contribution of the correction terms r (see (10)) is $r \simeq 0.451/L_c$.

In Eqs.(7) and (11) we can use the main terms of decomposition only.

The main term in (7) gives after the integration over x the standard expression for the radiation length L_{rad} without influence of multiple scattering.

$$\frac{I}{\varepsilon} = \frac{\alpha m^2}{4\pi\varepsilon_e} \left(1 + \frac{1}{9L_1} - \frac{4\pi}{15} \frac{\varepsilon}{\varepsilon_e} \right) \simeq L_{rad}^{-1} \left(1 - \frac{4\pi}{15} \frac{\varepsilon}{\varepsilon_e} \right), \\ \frac{1}{L_{rad}} = \frac{2Z^2\alpha^3 n_a L_1}{m^2} \left(1 + \frac{1}{9L_1} \right) = \frac{1}{L_{rad}^0} \left(1 + \frac{1}{9L_1} \right) \quad (13)$$

The integration over x of the main term in (11) gives (terms $\propto \sqrt{\varepsilon_e/\varepsilon}$ in the square brackets are neglected)

$$I_0 \simeq \frac{9\pi Z^2 \alpha^3 n_a \sqrt{\varepsilon \varepsilon_e}}{4\sqrt{2} m^2} L_1 \left[1 + \frac{1}{4L_1} \left(\ln \frac{\varepsilon}{\varepsilon_e} - \frac{46}{27} \right) + r_0 \right], \quad (14)$$

where $r_0 = (\ln 2 - C + \pi/4)/2L_1$. The corrections (without terms $\propto 1/L_1$) to (14) are calculated in Appendix B of ¹³(see Eq.(B.11)). The complete result is

$$\frac{I}{\varepsilon L_{rad}} \simeq \frac{5}{2} \sqrt{\frac{\varepsilon_e}{\varepsilon}} \left[1 - 2.37 \sqrt{\frac{\varepsilon_e}{\varepsilon}} - 4.57 \frac{\varepsilon_e}{\varepsilon} + \frac{1}{4L_1} \left(\ln \frac{\varepsilon}{\varepsilon_e} - 0.3455 \right) \right] \quad (15)$$

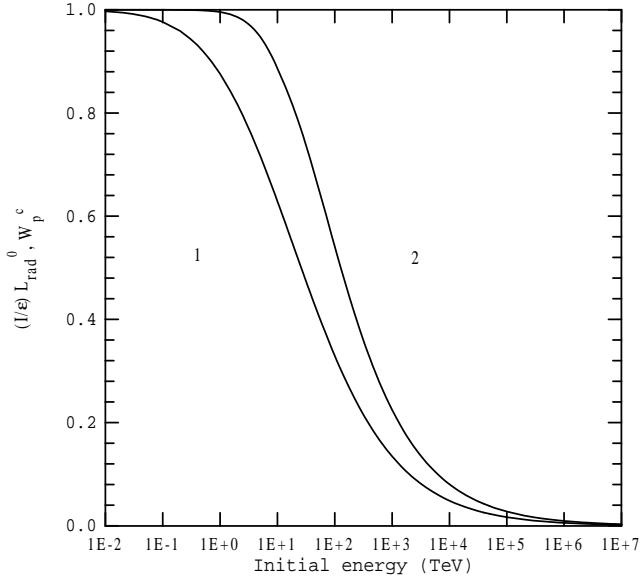


Figure 1. The relative energy losses of electron per unit time in terms of the Bethe-Maximon radiation length L_{rad}^0 : $\frac{I}{\varepsilon} L_{rad}^0$ in gold Eq.(12) vs the initial energy of electron (curve 1) and the total pair creation probability per unit time W_p^c (see Eq.(22)) in terms of the Bethe-Maximon total probability of pair creation W_{p0}^{BH} in gold vs the initial energy of photon (curve 2).

Although the coefficients in the last expression are rather large at two first terms of the decomposition over $\sqrt{\varepsilon_e/\varepsilon}$ this formula has the accuracy of the order of 10% at $\varepsilon \sim 10\varepsilon_e$.

The integral probability of radiation in terms of the Bethe-Maximon radiation length can be obtained from Eq.(12) dividing the integrand in all integrals by x . It is given in Fig.2. It should be mentioned that the standard Bethe-Maximon integral probability doesn't exist at all (the integral over ω has the logarithmic divergence at $\omega \rightarrow 0$). Due to the LPM effect the soft part of the spectrum is damped and integral over ω exists.

The integral probability of radiation for $\varepsilon \ll \varepsilon_e$ was calculated in ¹¹:

$$W = \frac{4}{3L_{rad}^0} \left(\ln \frac{\varepsilon_e}{\varepsilon} + C_2 \right),$$

$$C_2 = 2C - \frac{5}{8} + 12 \int_0^\infty \ln z \left(\frac{1}{z^3} - \frac{\cosh z}{\sinh^3 z} \right) dz \simeq 1.96 \quad (16)$$

In the case $\varepsilon \gg \varepsilon_e$ we can calculate the integral probability of radiation starting with Eq.(11). Conserving the main term, dividing it by $x\varepsilon$ and integrating over x we find

$$W_0 = \frac{11\pi Z^2 \alpha^3 n_a}{2\sqrt{2}m^2} \sqrt{\frac{\varepsilon_e}{\varepsilon}} L_1 \left[1 + \frac{1}{4L_1} \left(\ln \frac{\varepsilon}{\varepsilon_e} + \frac{8}{11} \right) + r_0 \right] \quad (17)$$

The correction terms to Eq.(16) are calculated in Appendix B of ¹³(see Eq.(B.13)). Substituting them we have

$$W = \frac{11\pi Z^2 \alpha^3 n_a}{2\sqrt{2}m^2} \sqrt{\frac{\varepsilon_e}{\varepsilon}} L_1 \left[1 - 1.23 \sqrt{\frac{\varepsilon_e}{\varepsilon}} + 1.65 \frac{\varepsilon_e}{\varepsilon} + \frac{1}{4L_1} \left(\ln \frac{\varepsilon}{\varepsilon_e} + 2.53 \right) \right]. \quad (18)$$

Ratio of the main terms of Eqs.(15) and (18) gives the mean energy of radiated photon

$$\bar{\omega} = \frac{9}{22} \varepsilon \simeq 0.409 \varepsilon. \quad (19)$$

3 Influence of multiple scattering on pair creation process

The probability of the pair creation by a photon can be obtained from the probability of the bremsstrahlung with help of the substitution law:

$$\omega^2 d\omega \rightarrow \varepsilon^2 d\varepsilon, \quad \omega \rightarrow -\omega, \quad \varepsilon \rightarrow -\varepsilon, \quad (20)$$

where ω is the initial photon energy, ε is the energy of the created electron. Making this substitution in Eq.(2) we obtain the spectral distribution of the pair creation probability (over the energy of the electron)

$$\begin{aligned} \frac{dW_p^c}{d\varepsilon} &= \frac{\alpha m^2}{2\pi\varepsilon\varepsilon'} \text{Im} \left[\Phi_p(\nu) - \frac{1}{L_c} F_p(\nu) \right], \\ \Phi_p(\nu) &= \nu \int_0^\infty dt e^{-it} \left[s_1 \left(\frac{1}{\sinh z} - \frac{1}{z} \right) - i\nu s_2 \left(\frac{1}{\sinh^2 z} - \frac{1}{z^2} \right) \right] \\ &= s_1 \left(\ln p - \psi \left(p + \frac{1}{2} \right) \right) + s_2 \left(\psi(p) - \ln p + \frac{1}{2p} \right), \\ F_p(\nu) &= \int_0^\infty \frac{dz e^{-it}}{\sinh^2 z} [s_1 f_1(z) - 2is_2 f_2(z)], \\ s_1 &= 1, \quad s_2 = \frac{\varepsilon^2 + \varepsilon'^2}{\omega^2}, \quad \varepsilon' = \omega - \varepsilon. \end{aligned} \quad (21)$$

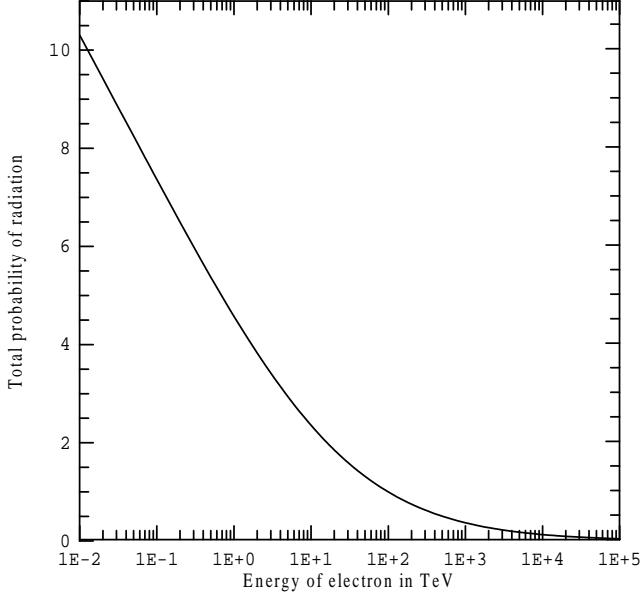


Figure 2. The total probability of photon emission W_0 in terms of the Bethe-Maximon radiation length L_{rad}^0 in gold vs the initial energy electron .

All entering functions are defined in (3).

The total probability of pair creation in the logarithmic approximation can be presented as (see (21))

$$\frac{W_p^c}{W_{p0}^{BH}} = \frac{9}{14} \frac{\omega_e}{\omega} \text{Im} \int_0^1 \frac{dy}{y(1-y)} \left[\left(\ln p - \psi \left(p + \frac{1}{2} \right) \right) + \left(1 - 2y + 2y^2 \right) \left(\psi(p) - \ln p + \frac{1}{2p} \right) \right], \quad p = \frac{bs}{4}, \quad (22)$$

where

$$s = \frac{1}{\sqrt{y(1-y)}}, \quad b = \exp \left(i \frac{\pi}{4} \right) \sqrt{\frac{L_1 \omega_e}{L_c \omega}}, \quad \omega_e = m \left(2\pi Z^2 \alpha^2 n_a \lambda_c^3 L_1 \right)^{-1},$$

here W_{p0}^{BH} is the Bethe-Maximon probability of pair photoproduction in the logarithmic approximation. Note that ω_e is four times larger than ε_e , in gold $\omega_e = 10.5$ TeV. This is just the value of photon energy starting with the

LPM effect becomes essential for the pair creation process in heavy elements. The total probability of pair creation W_p^c in gold is given in Fig.1 (curve 2), it is reduced by 10% at $\omega \simeq 9$ TeV and it cuts in half at $\omega \simeq 130$ TeV.

4 Conclusion

In this paper we considered the influence of multiple scattering on the bremsstrahlung process at any energy including the high-energy region ($\varepsilon \geq \varepsilon_e$), where all the spectrum of radiation is distorted. In this region the total intensity of radiation diminishes and respectively the radiation length increases. The cross section of e^-e^+ pair creation by a photon changes essentially if the photon energy $\omega \geq \omega_e = 4\varepsilon_e$, see Eq.(4).

If we restrict to the main terms of the decomposition Eq.(15) in asymptotic region $\varepsilon \gg \varepsilon_e$, then the intensity of radiation and the corresponding radiation length can be written as

$$I \simeq \frac{9}{16} \sqrt{\frac{\pi}{2}} Z \alpha^2 (\varepsilon n_a \ln(9\pi Z^2 \alpha^2 \varepsilon n_a a_{s2}^4))^{1/2}, \quad L_{rad} = \frac{\varepsilon}{I(\varepsilon)}. \quad (23)$$

The integral cross section of radiation follows from the integral probability of radiation (18)

$$\sigma = \frac{W}{n_a} \simeq \frac{11}{8} \sqrt{\frac{\pi}{2}} \frac{Z \alpha^2}{\sqrt{\varepsilon n_a}} (\ln(100\pi Z^2 \alpha^2 \varepsilon n_a a_{s2}^4))^{1/2}. \quad (24)$$

We have from for the total probability of pair creation by a photon at $\omega \gg \omega_e$ and the corresponding cross section

$$W_p \simeq \frac{3}{4} \sqrt{\frac{\pi}{2}} Z \alpha^2 \left(\frac{n_a}{\omega} \ln(2\pi Z^2 \alpha^2 \omega n_a a_{s2}^4) \right)^{1/2}, \quad \sigma_p = \frac{W_p}{n_a} \quad (25)$$

The Eqs.(23)-(25) don't depend on the electron mass and the cross sections of bremsstrahlung and pair creation diminish with energy and density n_a growth.

In this paper we considered the case of an infinitely thick target where the formation length is much shorter than the thickness of a target. Because of this we neglected the boundary effects. These effects were considered in detail in ^{8,10}, they can give quite essential contribution in the soft part of spectrum depending on the target thickness. We neglected also by effects of the polarization of a medium. They were considered in detail in ⁸. The relative contribution of polarization of a medium into probability of pair creation is discussed in ¹⁵

$$\frac{\omega_0^2 \varepsilon \varepsilon'}{\omega^2 m^2} \leq \frac{\omega_0^2}{m^2} < 10^{-7} \ll 1, \quad \omega_0^2 = \frac{4\pi e^2 n_e}{m}, \quad (26)$$

where n_e is the number density of electron in the medium, ω_0 is the plasma frequency. The contribution of polarization of a medium into the total energy losses in thick target is of the order ω_0/m . The polarization of a medium affects at the soft part of the spectrum only at $\omega \leq \omega_p = \gamma\omega_0$ ($x \leq \omega_p/\varepsilon = \omega_0/m$). Even for heavy elements $\omega_0/m \sim 2 \cdot 10^{-4}$. This contribution was analyzed in⁸.

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